Deep Learning for Time Series

Session 2: ConvNets and Recurrent architectures

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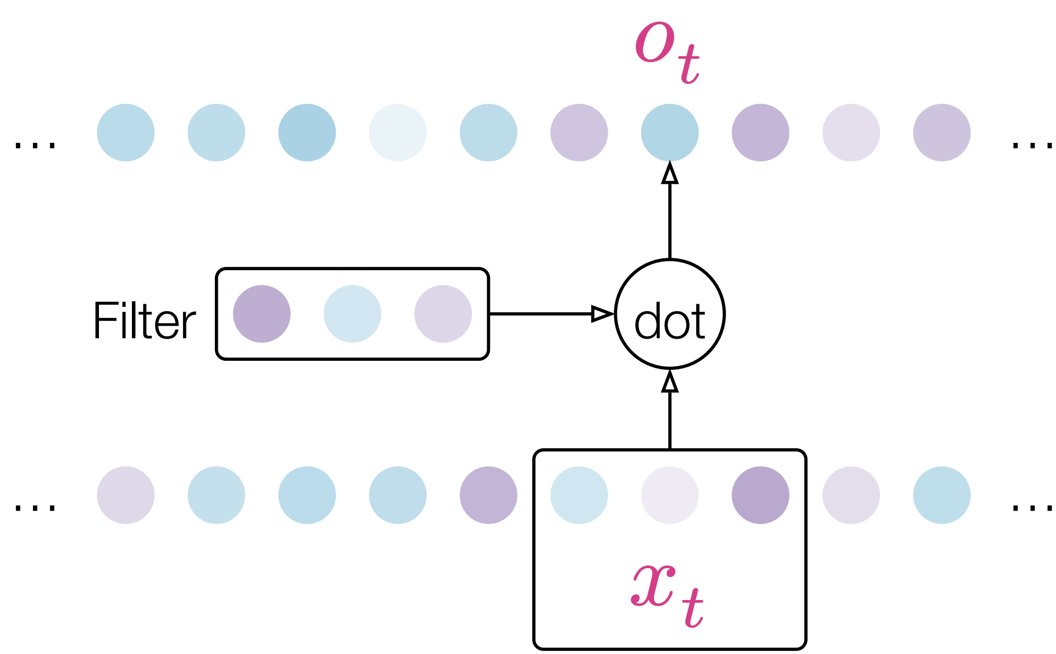
Convolutional architectures

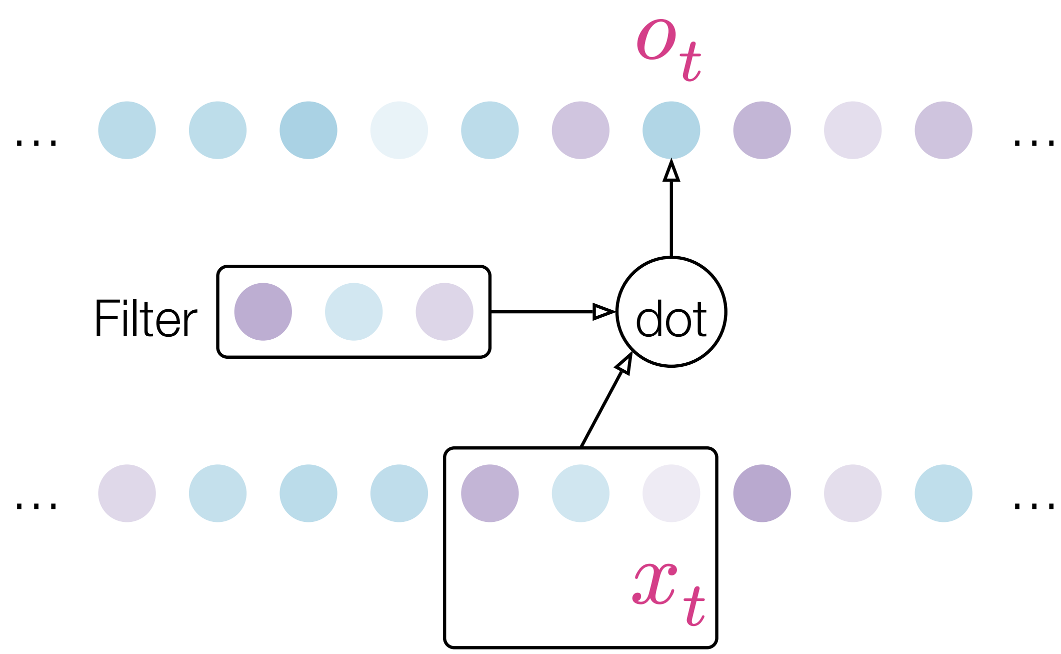
ConvNets 101

- Basic time series processing: 1d convolutions (over time)
- Limited receptive field: co-localization matters



Causal convolutions

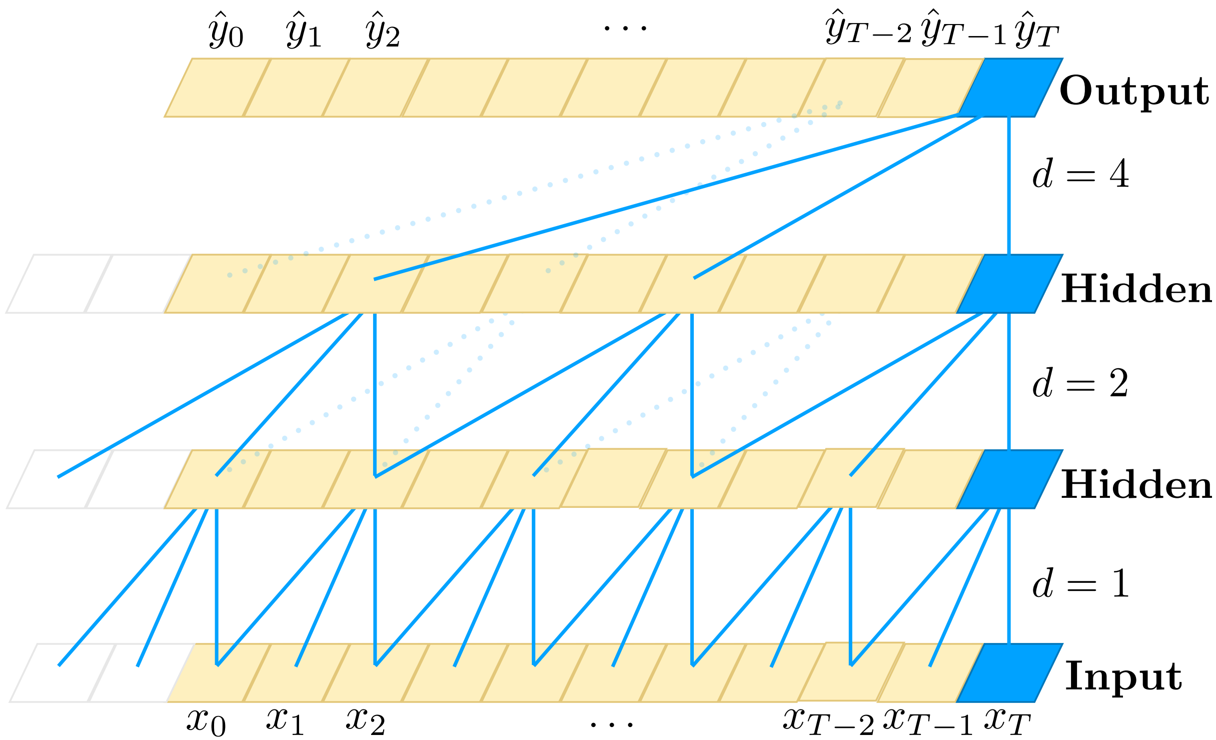
- Forecasting tasks: cannot access the future
- Causal convolution: convolve on past information alone (asymmetric window)



Temporal Convolution Network (TCN)

- Main idea: cascade dilated causal convolutions

  $\Rightarrow$ Larger receptive field

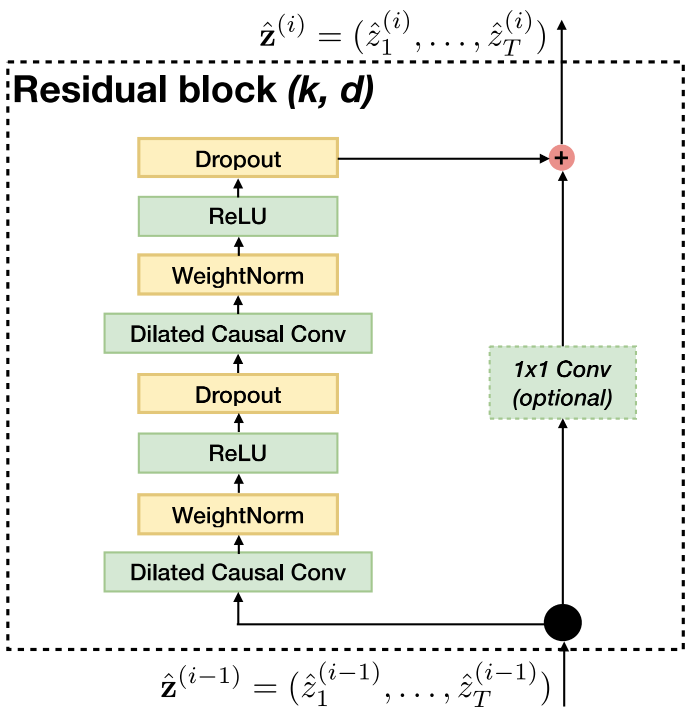


Source: “An Empirical Evaluation of Generic Convolutional and Recurrent Networks for Sequence Modeling”, Bai et al., arXiV 2018

Temporal Convolution Network

- Additional improvements:
  - Residual connections

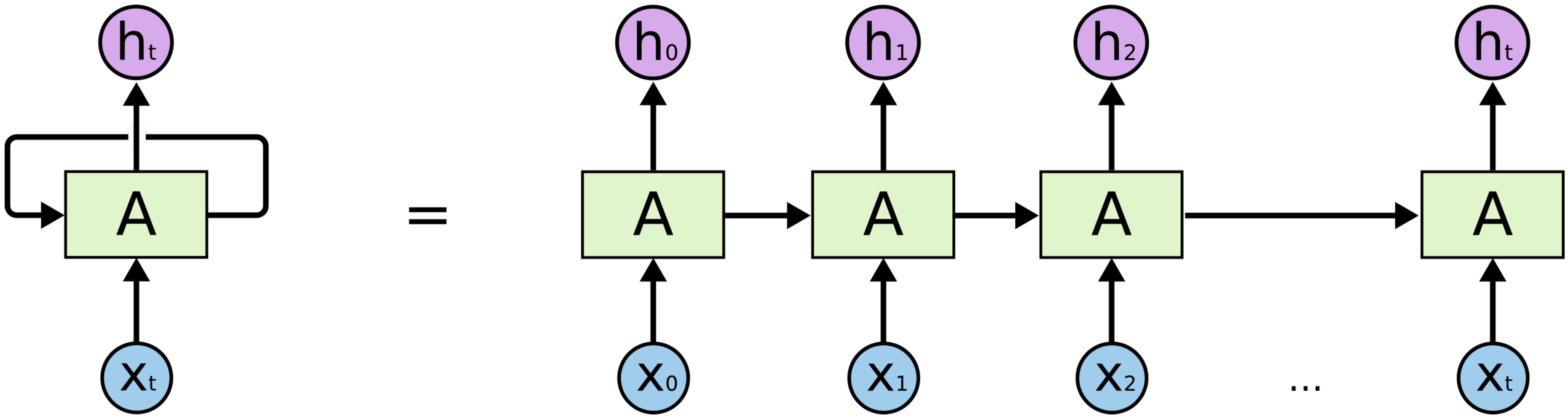
    $\Rightarrow$ Multi-resolution analysis
  - Normalization+Dropout layers



Source: "An Empirical Evaluation of Generic Convolutional and Recurrent Networks for Sequence Modeling", Bai et al., arXiV 2018

Recurrent architectures

- Very flexible model (any length, let the model learn its memory needs, ...)

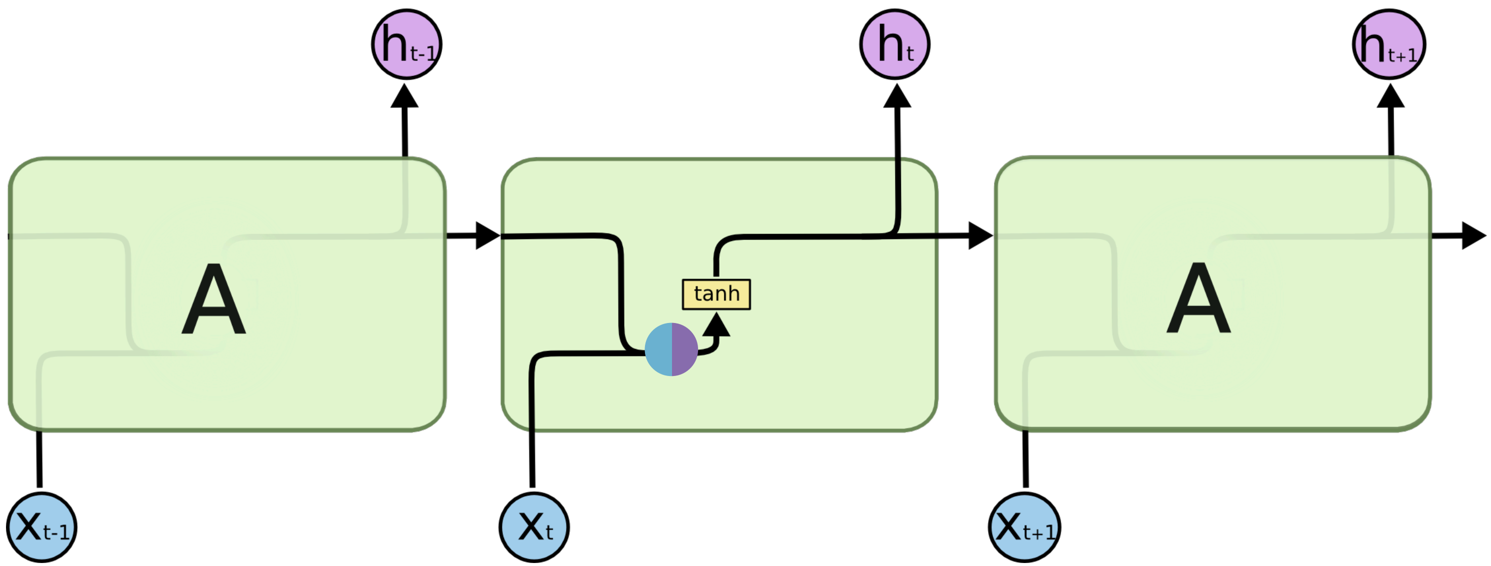


Source: Christopher Olah's blog

"Vanilla" RNN

- Hidden state is computed as:

$$h_t = \varphi(\bullet)$$

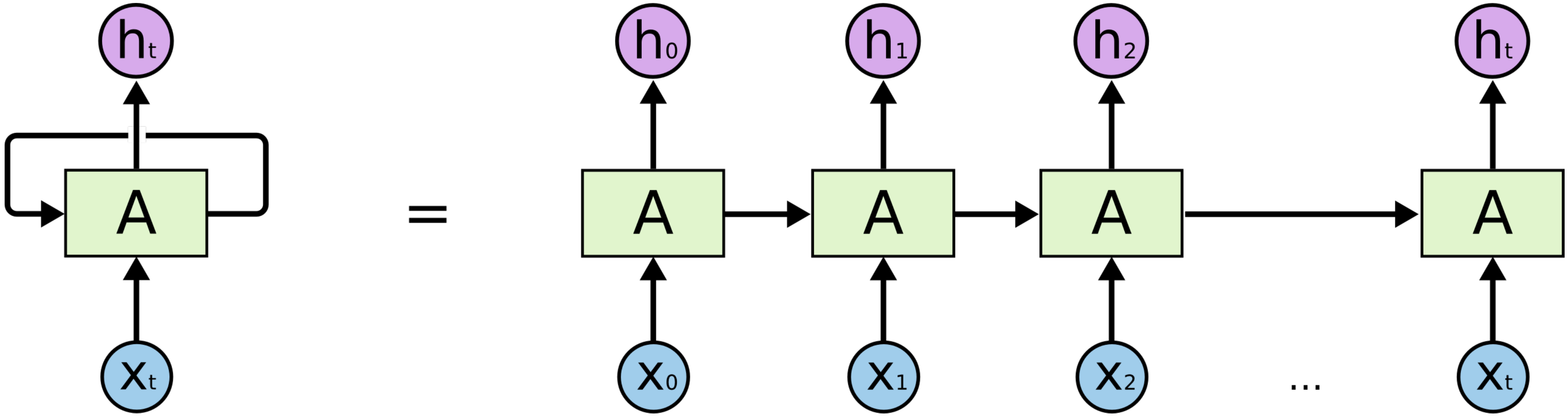


Source: Christopher Olah's blog

x_t h_{t-1} Linear combination of x_t and h_{t-1}

"Vanilla" RNN properties

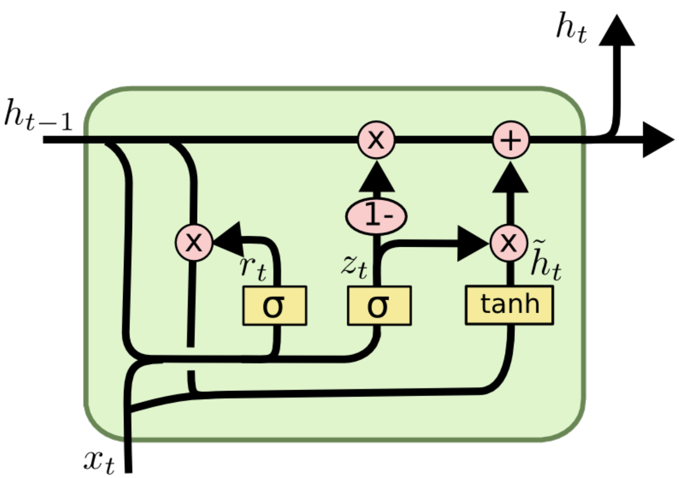
- Very flexible model (any length, let the model learn its memory needs, …)
- Difficult to learn in practice
  - Slow (lack of parallelism)
  - Vanishing gradients (hard to learn long-term dependencies) or exploding gradients (if φ is unbounded)



Source: Christopher Olah's blog

Gated Recurrent Unit (GRU)

- At each time step, keep only part of the information
  - Through **gating mechanism**



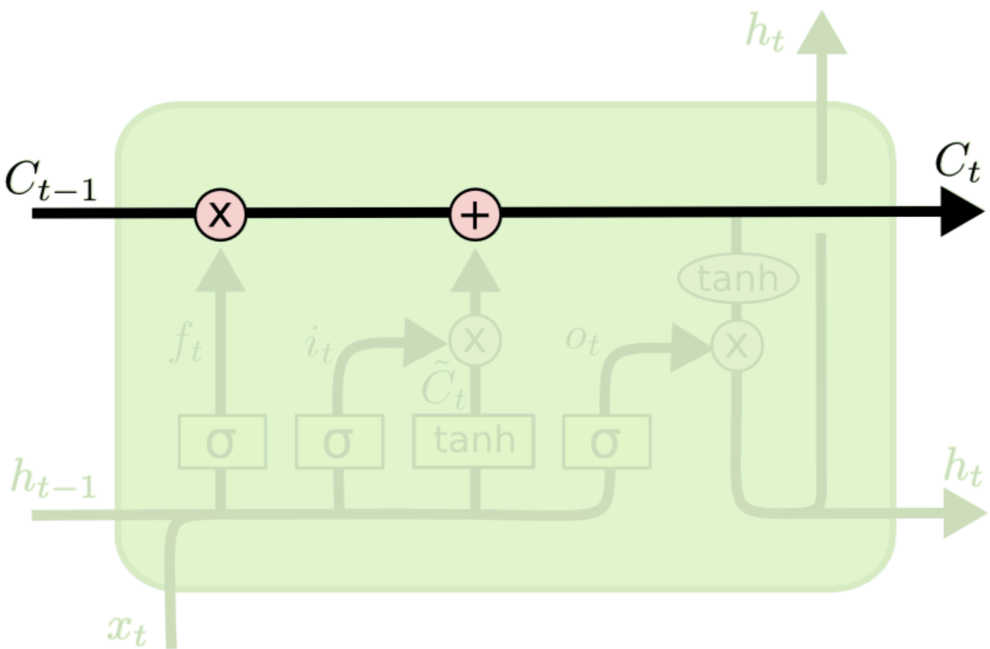
Source: Christopher Olah's blog

$$z_t = \sigma(\cdot) \quad \text{(update gate)}$$
$$r_t = \sigma(\cdot) \quad \text{(reset gate)}$$
$$\tilde{h}_t = \varphi(W \cdot x_t + R \cdot [r_t \odot h_{t-1}])$$
$$h_t = (1 - z_t) \odot h_{t-1} + z_t \odot \tilde{h}_t$$

x_t h_{t-1} Linear combination of x_t and h_{t-1}

Long Short Term Memory (LSTM)

- Similar ideas as in GRUs, but:
  - an additional *cell state* C_t

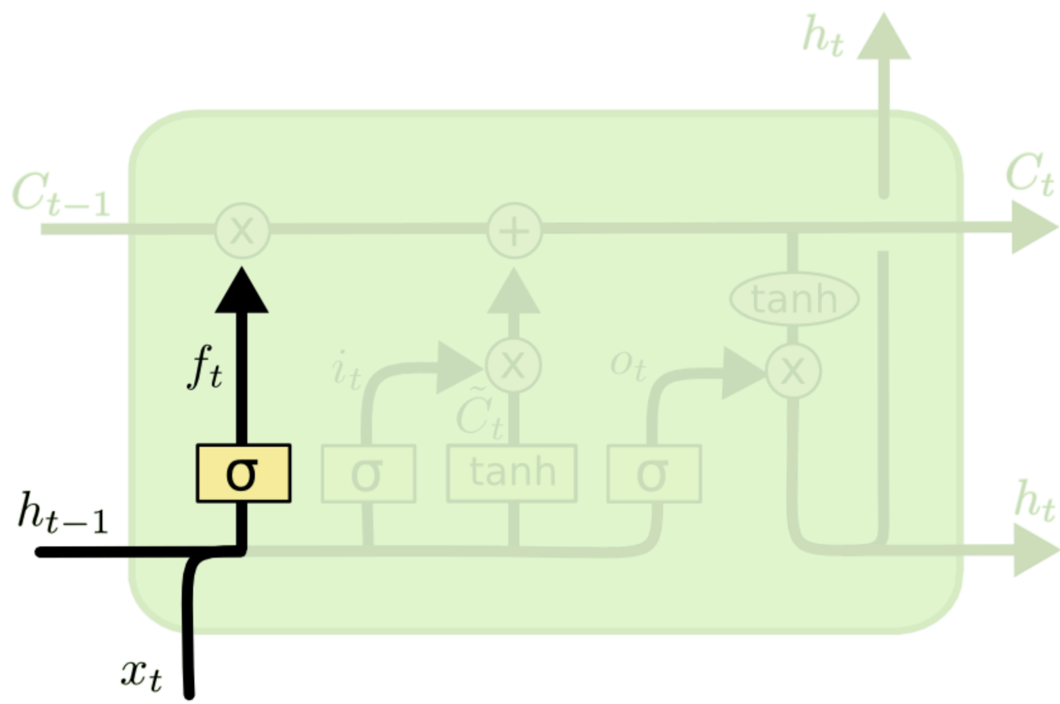


Source: Christopher Olah's blog

  - input and forget gates are made independent (in place of z_t in GRU)

Long Short Term Memory (LSTM)

- **Forget gate**: $f_t = \sigma(\,\cdot\,)$

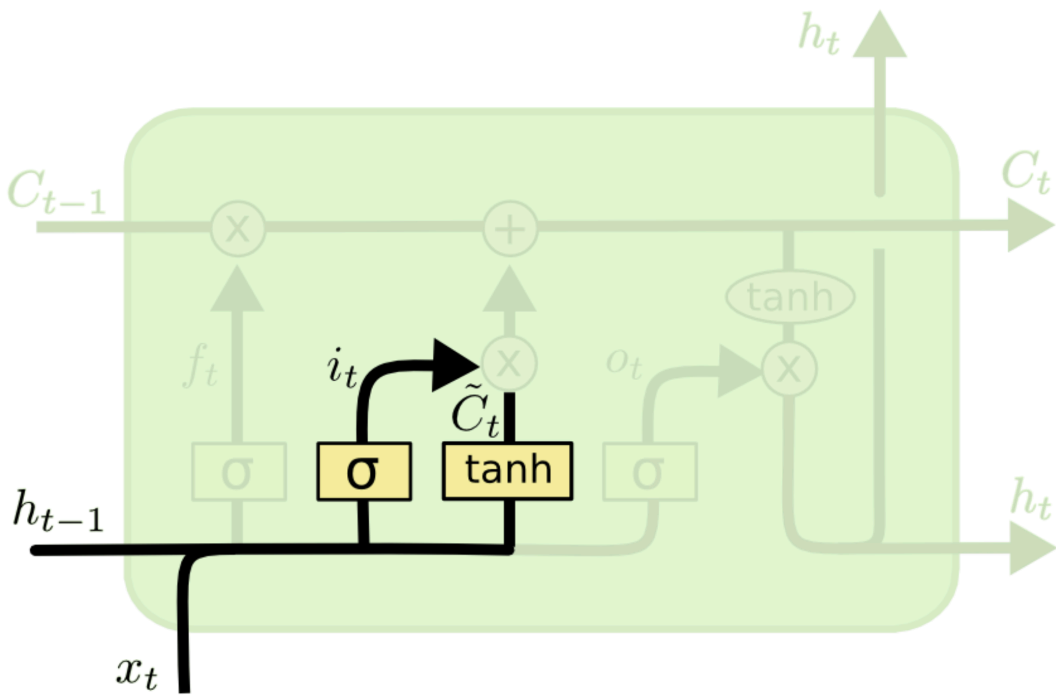


Source: Christopher Olah's blog

x_t h_{t-1} Linear combination of x_t and h_{t-1}

Long Short Term Memory (LSTM)

- **Input gate**: $i_t = \sigma(\ )$
- **Suggested C_t update**: $\tilde{C}_t = \varphi(\ )$

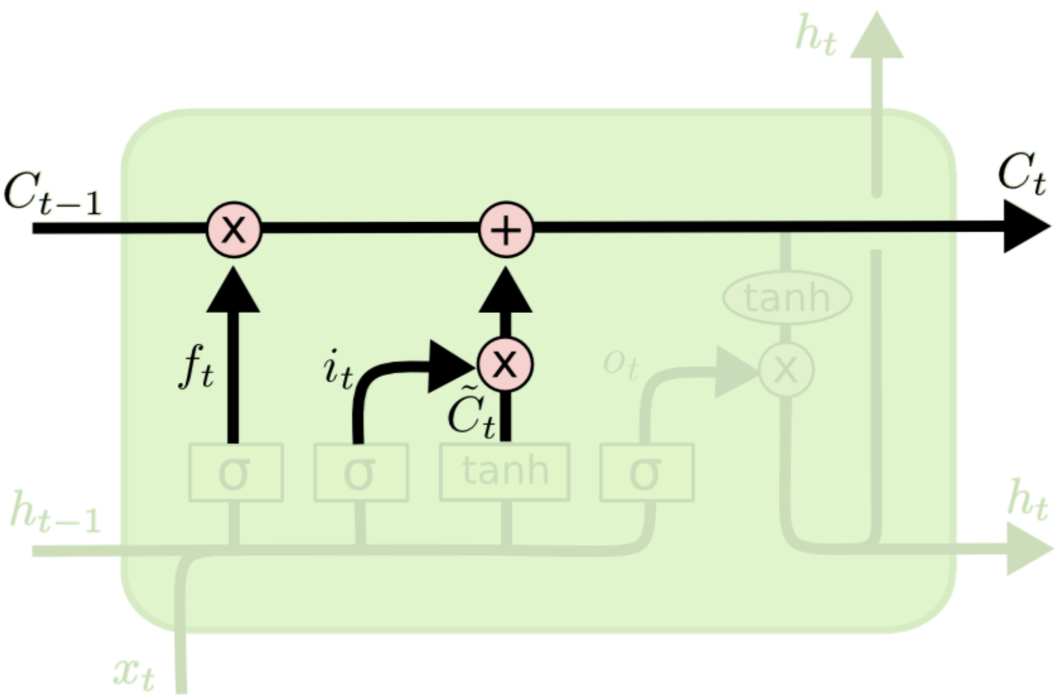


Source: Christopher Olah's blog

x_t h_{t-1} Linear combination of x_t and h_{t-1}

Long Short Term Memory (LSTM)

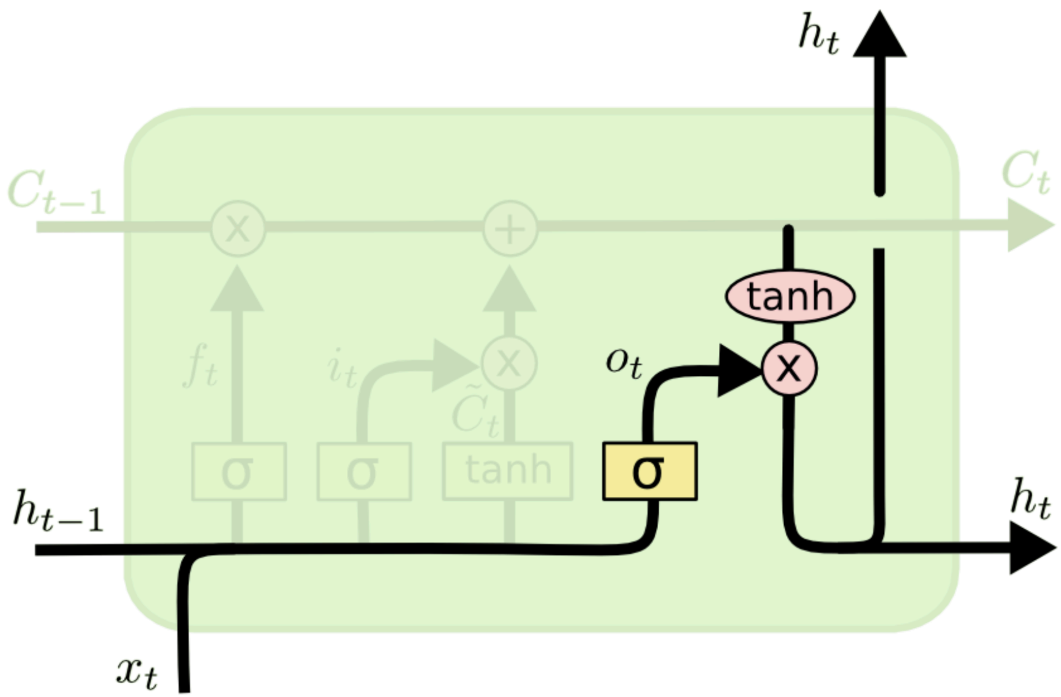
- **C_t update rule**: $C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$



Source: Christopher Olah's blog

Long Short Term Memory (LSTM)

- **Output gate**: $o_t = \sigma(\text{◒})$
- **Hidden state update rule**: $h_t = o_t \odot \varphi(C_t)$

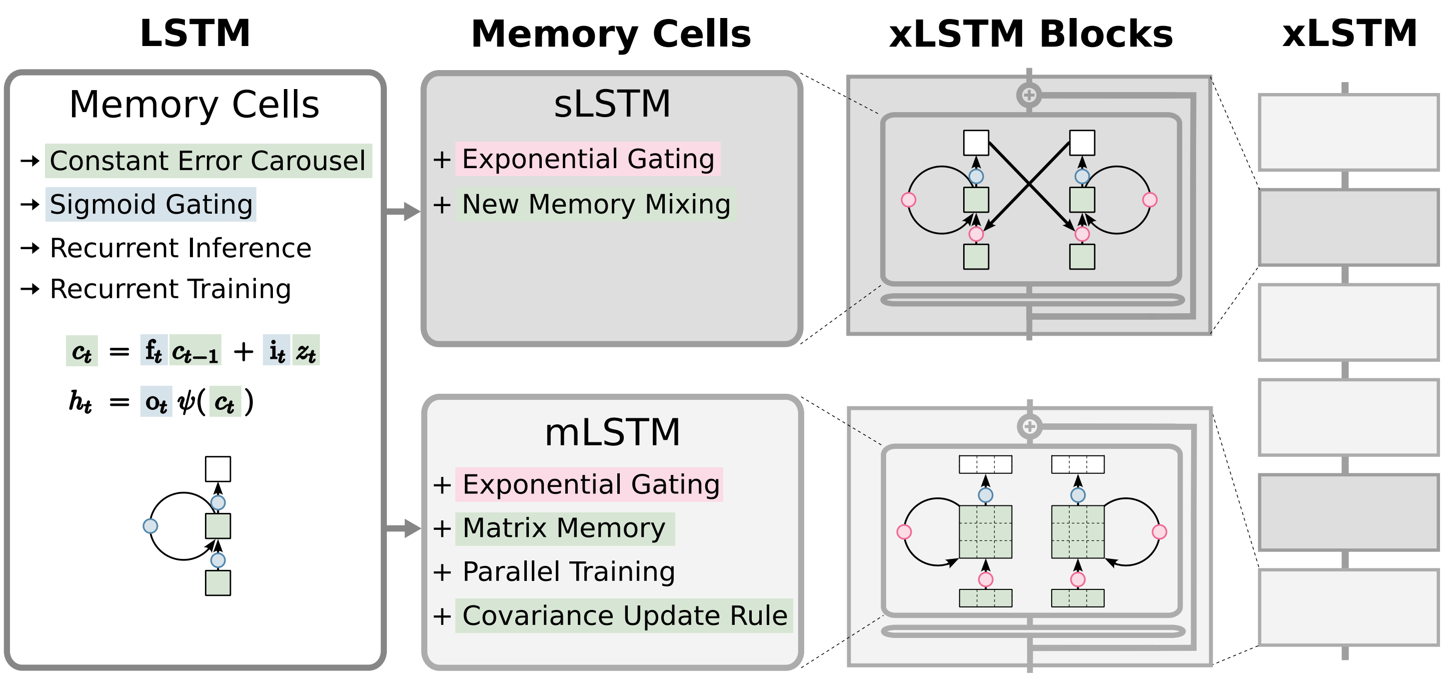


Source: Christopher Olah's blog

● x_t ● h_{t-1} ◐ Linear combination of x_t and h_{t-1}

xLSTM

- A “modern” LSTM variant
  - Made of sLTSM and mLSTM layers
  - Embedded in blocks with normalization layers, residual connections, *à la* Transformer



Source: “xLSTM: Extended Long Short-Term Memory” by Beck et al., NeurIPS 2024

xLSTM

- What's "new"?
  - In both sLSTM and mLSTM layers:
    - Exponential activation (to face vanishing gradients)
  - In sLTSM only:
    - Multi-head
  - In mLSTM only:
    - Novel memory store
    - Drop recurrence for gate computations: better parallelism

xLSTM: focus on sLSTM layers

- Exponential activation for input and forget gates:

$$i_t = \exp(\ )$$
$$f_t = \max(\exp(\ ), \sigma(\ ))$$

$\Rightarrow$ Need normalization:

$$n_t = f_t \odot n_{t-1} + i_t$$
$$h_t = o_t \odot C_t \oplus n_t$$

- Multi-head: keep separate linear combinations per head

x_t h_{t-1} Linear combination of x_t and h_{t-1}

xLSTM: focus on mLSTM layers

- Exponential activation as in sLSTM
- Memory store

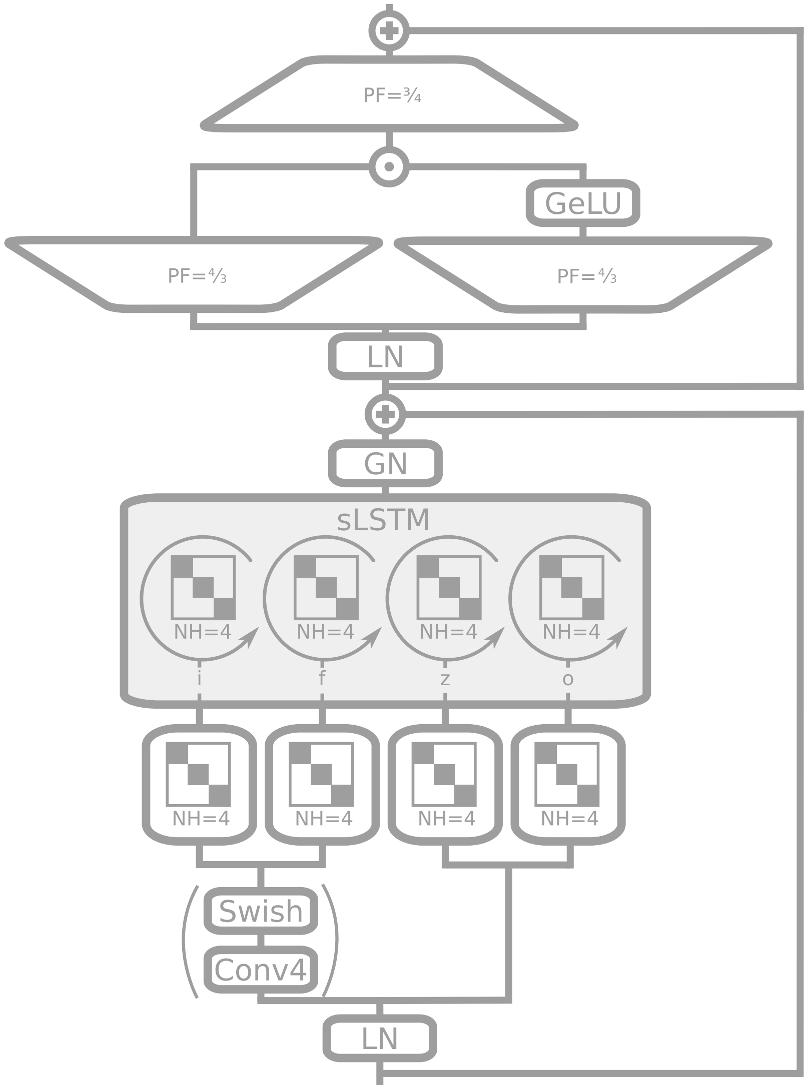
$$
\begin{aligned}
C_t &= f_t \odot C_{t-1} + i_t \odot v_t k_t^\top \\
\tilde{h}_t &= C_t q_t && \text{(up to normalization)}
\end{aligned}
$$

  - Simplified case (no gate): similar to QKV in self-attention
- Drop recurrence for gate computations: better parallelism

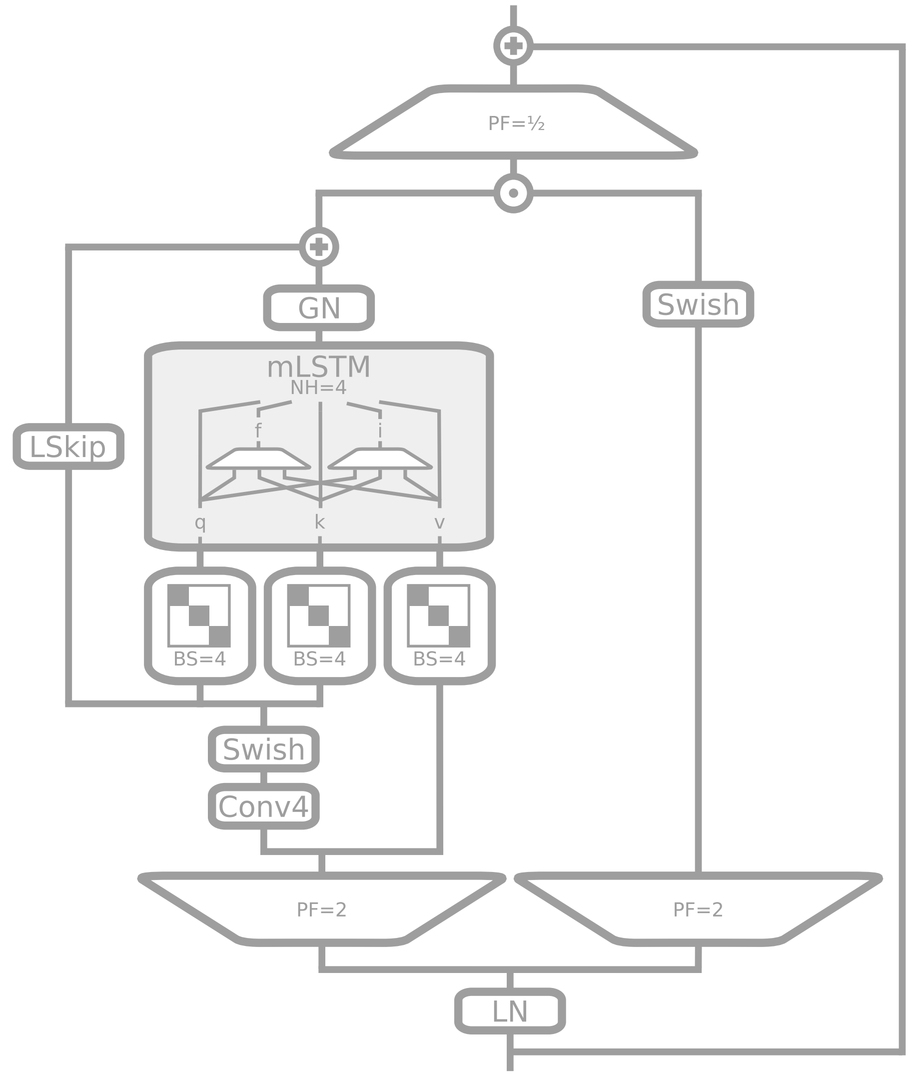
$$
\begin{aligned}
i_t &= \exp(\bullet) \\
f_t &= \max(\exp(\bullet), \sigma(\bullet)) \\
o_t &= \sigma(\bullet)
\end{aligned}
$$

● x_t ● h_{t-1} ◐ Linear combination of x_t and h_{t-1}

xLSTM: Building blocks



An sLSTM block



An mLSTM block